Accounting for running α_s for the non-singlet components of the structure functions F_1 and g_1 at small x.

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Infrared evolution equations incorporating the running QCD coupling are constructed and solved for the non-singlet structure functions f_{NS}. Accounting for dropped logs of x in DGLAP it leads to a scaling-like small x behaviour $f_{NS} \sim (\sqrt{Q^2}/x)^a$. In contrast to the leading logarithmic approximation, intercepts a are numbers and do not contain α_s. It is also shown that the leading logarithmic approximation may be unreliable for predicting Q^2 -dependence of the DIS structure functions in the HERA range.

Non-singlet structure functions, i.e. flavour-dependent contributions to the deep inelastic structure functions, have been the object of intensive theoretical investigation. First, they are interesting because they are experimentally measurable quantities; second, they are comparatively technically simple for analysis, and can be regarded as a starting ground for a theoretical description of DIS structure functions. In the present talk we discuss the explicit expressions[3] for the non-singlet contribution f^+_{NS} to the structure function F_1 and for the non-singlet contribution f^-_{NS} to the spin structure function g_1 at x. These expressions account for both leading (double-logarithmic) and subleading (single-logarithmic) contributions to all orders in QCD coupling and include the running α_s effects. Contrary to DGLAP[1] and to some other works on small x, we do not use a priori the standard parametrisation $\alpha_s = \alpha_s(Q^2)$ in our evolution eqs. Indeed it has been shown recently [3] that such a dependence is a good approximation at large x but is not correct when x is small.

As we account for double-logarithmic (DL) and single-logarithmic (SL) contributions to all orders and regardless of the arguments, we cannot use the DGLAP eqs. Instead, we construct and solve two-dimensional infrared evolution equations (IREE) for f_{NS} appreciating evolution with respect to x and to Q^2. In the context of this method, $f^{\pm}_{NS}$ evolves with respect to the infrared cut-off μ in the transverse momentum space: $k_{i\perp} > \mu$ for all virtual particles. In doing so, we provide $f^{\pm}_{NS}(x, Q^2)$ with μ dependence too. However,

it's unavoidable when α_s is running because the standard expression

$$\alpha_s(t) = \frac{1}{b\ln(t/\Lambda^2_{QCD})} \ , \tag{1}$$

is valid only when $t \gg \Lambda^2_{QCD}$ and therefore if we introduce the infrared cut-off as

$$k_{i\perp} > \mu > m_{max} \gg \Lambda_{QCD} \ , \tag{2}$$

with m_{max} being the mass of the heaviest involved quark, we can neglect quark masses and still do not have infrared singularities. Besides the restrictions imposed by Eq. (2) μ is not fixed, so f_{NS} can evolve with respect to μ, eventually arriving at the following expressions for the non-singlet structure functions:

$$f^{\pm}_{NS} = \int_{-\imath\infty}^{\imath\infty} \frac{d\omega}{2\pi\imath} C \left(\frac{1}{x}\right)^{\omega} \omega \exp\left([(1+\lambda\omega)F_0^{\pm}]y\right) \tag{3}$$

where C is an (non-perturbative) input and $F_0^{\pm}$ are the new anomalous dimensions. They account for the total resummation of the most essential at small x NLO contributions of the type $(\alpha_s/\omega^2)^n$ and $(\alpha_s/\omega)^n \quad (n = 1, ...)$,

$$F_0^{\pm} = 2\left[\omega - \sqrt{\omega^2 - (1+\lambda\omega)(A(\omega) + \omega D^{\pm}(\omega))/2\pi^2}\right] \tag{4}$$

where

$$A(\omega) = \frac{4C_F\pi}{b}\left[\frac{\eta}{\eta^2+\pi^2} - \int_0^{\infty} \frac{d\rho\exp(-\rho\omega)}{(\rho+\eta)^2+\pi^2}\right] \ . \tag{5}$$

and

$$D^{\pm}(\omega) = \frac{2C_F}{\omega b^2 N}\int_0^{\infty} d\rho \exp(-\rho\omega)\ln\left(\frac{\rho+\eta}{\eta}\right)\left[\frac{\rho+\eta}{(\rho+\eta)^2+\pi^2} \mp \frac{1}{\rho+\eta}\right] \ . \tag{6}$$

We have used in Eqs. (5,6) the following notations: $\eta = \ln(\mu^2/\Lambda^2_{QCD})$, $\rho = \ln(s/\mu^2)$, $\lambda = 1/2$ and the first coefficient of the β -function $b = (11N - 2n_f)/12\pi$. A corresponds to accounting for running α_s. π^2 in denominators appears due to analytical properties of $\alpha_s(t)$: it must have a non-zero imaginary part when t is time-like. D contains the signature-dependent contributions.

Expanding the resummed anomalous dimension $F_0^{\pm}$ into series in $1/\omega$ we reproduce the singular in ω terms of LO and NLO DGLAP- anomalous dimensions where $\alpha_s(Q^2)C_F/2\pi$ is replaced by A.

It is shown in Refs. [3] that A can be approximated by $\alpha_s(Q^2)C_F/2\pi$ only at large x. Concerning the small- x and large Q^2 asymptotics of $f^{\pm}_{NS}$, Eq. (3) reads that

$$f^{\pm}_{NS} \sim x^{-\omega_0^{\pm}}(Q^2/\mu^2)^{\omega_0^{\pm}/2}, \tag{7}$$

with the intercepts $\omega_0^{\pm}$ being the leading, i.e. the rightmost, singularities of $F_0^{\pm}$. Eqs. (3,4) read that $\omega_0^{\pm}$ are the rightmost roots of

$$\omega^2 - (1 + \lambda\omega)(A(\omega) + \omega D^{\pm}(\omega))/2\pi^2 = 0 . \qquad (8)$$

Eq. (8) contains n_f, Λ_{QCD} and μ as parameters. Choosing e.g. $n_f = 3$ and $\Lambda_{QCD} = 0.1$ GeV one can solve Eq. (8) numerically and obtain $\omega_0^{\pm}$ as a function of μ. The solutions are given in Fig. 1. Both ω_0^{+} and ω_0^{-} acquire imaginary parts at $\mu < 0.4$ GeV. As besides, for applicability of Eq. (1) μ must be much greater than Λ_{QCD}, we think that the region $\mu < 0.4$ GeV is beyond control of our approach. Both ω_0^{+} and ω_0^{-} are maximal at $\mu \approx 1$ Gev and slowly decrease with μ increasing. Therefore we can estimate values of the intercepts as

$$\Omega_0^{+} = 0.37, \quad \Omega_0^{-} = 0.4 . \qquad (9)$$

It is interesting that this result was independently confirmed[4] recently by extrapolating of fits for f_3 into small x region. Eq. (9) was obtained from Eq. (8) which contains π^2 -terms. Basically, they are beyond of control of logarithmic accuracy and might be dropped. With π^2 -terms dropped, we obtain the smooth curves for $\omega_0^{\pm}$ depicted in Fig. 1. These curves show that π^2 -terms can be easily neglected for the values of μ greater than $\mu_0 = 5.5$ GeV. However, $\mu_0^2 = 30$ Gev^2 corresponds to the HERA Q^2 range. Then Eq. (9) immediately implies that, with such a big μ_0, the logarithmic accuracy is not enough to obtain a correct Q^2 dependence in the HERA range. On the other hand, it also explains why DLA estimates $\alpha_s = \alpha_s(Q^2)$ may be correct for predicting the x dependence: indeed, in DLA where the coupling is fixed, one should use rather $\alpha_s = \alpha_s(\mu_0^2)$ than $\alpha_s = \alpha_s(Q^2)$ as taken from DGLAP, but as it happens that $\mu_0^2 = Q^2$ in the HERA range, both estimates coincide.

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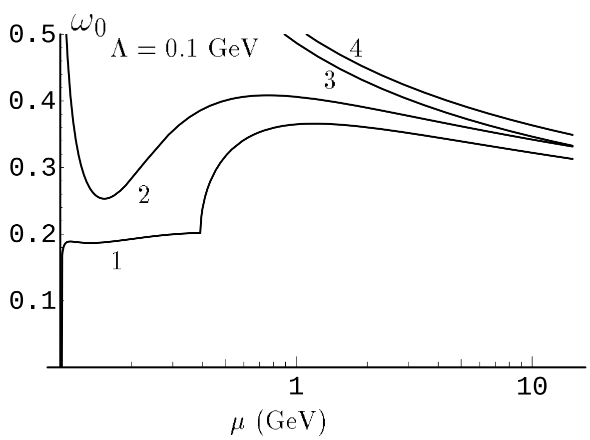

Figure 1: Dependence of the intercept ω_0 on infrared cutoff μ at $\Lambda_{QCD} = 0.1$ GeV: 1– for f_1^{NS}; 2– for g_1^{NS}; 3– and 4– for f_1^{NS} and g_1^{NS} respectively without account of π^2-terms.

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